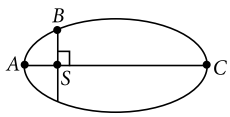
CHAPTER 8

Gravitation

8.2 Kepler's Laws

1. The kinetic energies of a planet in an elliptical orbit about the Sun, at positions A, B and C are K_A, K_B and K_C, respectively. AC is the major axis and SB is perpendicular to AC at the position of the Sun S as shown in the figure. Then

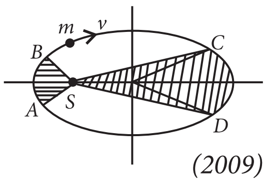


(a) $K_A < K_B < K_C$
(b) $K_A > K_B > K_C$
(c) $K_B < K_A < K_C$
(d) $K_B > K_A > K_C$ *(NEET 2018)*

2. A planet moving along an elliptical orbit is closest to the sun at a distance r_1 and farthest away at a distance of r_2. If v_1 and v_2 are the linear velocities at these points respectively, then the ratio $\frac{v_1}{v_2}$ is

(a) $(r_1/r_2)^2$ (b) r_2/r_1
(c) $(r_2/r_1)^2$ (d) r_1/r_2 *(2011)*

3. The figure shows elliptical orbit of a planet m about the sun S. The shaded area SCD is twice the shaded area SAB. If t_1 is the time for the planet to move from C to D and t_2 is the time to move from A to B then



(a) $t_1 = 4t_2$
(b) $t_1 = 2t_2$
(c) $t_1 = t_2$
(d) $t_1 > t_2$ *(2009)*

4. The period of revolution of planet A around the sun is 8 times that of B. The distance of A from the sun is how many times greater than that of B from the sun?

(a) 4 (b) 5 (c) 2 (d) 3 *(1997)*

5. The distance of two planets from the sun are 10^{13} m and 10^{12} m respectively. The ratio of time periods of the planets is

(a) $\sqrt{10}$ (b) $10\sqrt{10}$
(c) 10 (d) $1/\sqrt{10}$ *(1994, 1988)*

6. A planet is moving in an elliptical orbit around the sun. If T, V, E and L stand respectively for its kinetic energy, gravitational potential energy, total energy and magnitude of angular momentum about the centre of force, which of the following is correct ?

(a) T is conserved.
(b) V is always positive.
(c) E is always negative.
(d) L is conserved but direction of vector L changes continuously. *(1990)*

7. The largest and the shortest distance of the earth from the sun are r_1 and r_2. Its distance from the sun when it is at perpendicular to the major-axis of the orbit drawn from the sun is

(a) $\frac{r_1 + r_2}{4}$ (b) $\frac{r_1 + r_2}{r_1 - r_2}$
(c) $\frac{2r_1 r_2}{r_1 + r_2}$ (d) $\frac{r_1 + r_2}{3}$ *(1988)*

8.3 Universal Law of Gravitation

8. Two astronauts are floating in gravitational free space after having lost contact with their spaceship. The two will

(a) move towards each other
(b) move away from each other
(c) will become stationary
(d) keep floating at the same distance between them. *(NEET 2017)*

9. Kepler's third law states that square of period of revolution (T) of a planet around the sun, is proportional to third power of average distance r between sun and planet *i.e.* $T^2 = Kr^3$ here K is constant. If the masses of sun and planet are M and m respectively then as per Newton's law of gravitation force of attraction between them is $F = \frac{GMm}{r^2}$, here G is gravitational constant. The relation between G and K is described as

(a) $K = G$ (b) $K = \frac{1}{G}$
(c) $GK = 4\pi^2$ (d) $GMK = 4\pi^2$
(2015 Cancelled)

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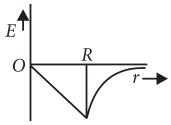
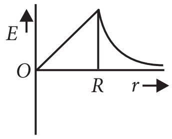
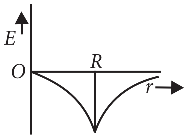
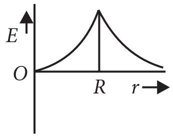
10. Dependence of intensity of gravitational field (E) of earth with distance (r) from centre of earth is correctly represented by

(a) 

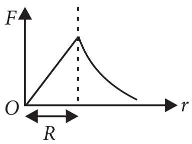
(b) 

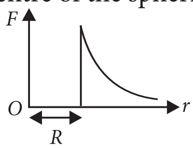
(c) 

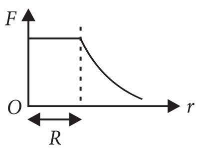
(d) 

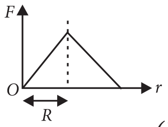
(2014)

11. Which one of the following plots represents the variation of gravitational field on a particle with distance r due to a thin spherical shell of radius R? (r is measured from the centre of the spherical shell)

(a) 

(b) 

(c) 

(d) 

(Mains 2012)

12. Two spheres of masses m and M are situated in air and the gravitational force between them is F. The space around the masses is now filled with a liquid of specific gravity 3. The gravitational force will now be
(a) $3F$ (b) F
(c) $F/3$ (d) $F/9$ *(2003)*

13. Gravitational force is required for
(a) stirring of liquid (b) convection
(c) conduction (d) radiation *(2000)*

14. A body of weight 72 N moves from the surface of earth at a height half of the radius of earth, then gravitational force exerted on it will be
(a) 36 N (b) 32 N
(c) 144 N (d) 50 N *(2000)*

15. Two particles of equal mass m go around a circle of radius R under the action of their mutual gravitational attraction. The speed v of each particle is
(a) $\frac{1}{2}\sqrt{\frac{Gm}{R}}$ (b) $\sqrt{\frac{4Gm}{R}}$
(c) $\frac{1}{2R}\sqrt{\frac{1}{Gm}}$ (d) $\sqrt{\frac{Gm}{R}}$ *(1995)*

16. The earth (mass $= 6 \times 10^{24}$ kg) revolves around the sun with an angular velocity of 2×10^{-7} rad/s in a circular orbit of radius 1.5×10^{8} km. The force exerted by the sun on the earth, in newton, is
(a) 36×10^{21} (b) 27×10^{39}
(c) zero (d) 18×10^{25} *(1995)*

17. If the gravitational force between two objects were proportional to $1/R$ (and not as $1/R^2$), where R is the distance between them, then a particle in a circular path (under such a force) would have its orbital speed v, proportional to
(a) R (b) R^0 (independent of R)
(c) $1/R^2$ (d) $1/R$ *(1994, 1989)*

8.5 Acceleration due to Gravity of the Earth

18. If the mass of the Sun were ten times smaller and the universal gravitational constant were ten times larger in magnitude, which of the following is not correct?
(a) Raindrops will fall faster.
(b) Walking on the ground would become more difficult.
(c) Time period of a simple pendulum on the Earth would decrease.
(d) g on the Earth will not change. *(NEET 2018)*

19. A spherical planet has a mass M_p and diameter D_p. A particle of mass m falling freely near the surface of this planet will experience an acceleration due to gravity, equal to
(a) $\frac{4GM_p}{D_p^2}$ (b) $\frac{GM_p m}{D_p^2}$
(c) $\frac{GM_p}{D_p^2}$ (d) $\frac{4GM_p m}{D_p^2}$ *(2012)*

20. Imagine a new planet having the same density as that of earth but it is 3 times bigger than the earth in size. If the acceleration due to gravity on the surface of earth is g and that on the surface of the new planet is g', then
(a) $g' = g/9$ (b) $g' = 27g$
(c) $g' = 9g$ (d) $g' = 3g$ *(2005)*

21. The density of a newly discovered planet is twice that of earth. The acceleration due to gravity at the surface of the planet is equal to that at the surface of the earth. If the radius of the earth is R, the radius of the planet would be
(a) $2R$ (b) $4R$ (c) $\frac{1}{4}R$ (d) $\frac{1}{2}R$
(2004)

22. The acceleration due to gravity on the planet A is 9 times the acceleration due to gravity on planet B. A man jumps to a height of 2 m on the surface of A. What is the height of jump by the same person on the planet B ?
(a) (2/9) m (b) 18 m
(c) 6 m (d) (2/3) m *(2003)*

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23. What will be the formula of mass of the earth in terms of g, R and G ?

(a) $G\frac{R}{g}$ (b) $g\frac{R^2}{G}$ (c) $g^2\frac{R}{G}$ (d) $G\frac{g}{R}$. *(1996)*

24. The acceleration due to gravity g and mean density of the earth ρ are related by which of the following relations? (where G is the gravitational constant and R is the radius of the earth.)

(a) $\rho = \frac{3g}{4\pi GR}$ (b) $\rho = \frac{3g}{4\pi GR^3}$

(c) $\rho = \frac{4\pi gR^2}{3G}$ (d) $\rho = \frac{4\pi gR^3}{3G}$ *(1995)*

25. The radius of earth is about 6400 km and that of mars is 3200 km. The mass of the earth is about 10 times mass of mars. An object weighs 200 N on the surface of earth. Its weight on the surface of mars will be

(a) 20 N (b) 8 N (c) 80 N (d) 40 N *(1994)*

8.6 Acceleration due to Gravity Below and Above the Surface of Earth

26. A body weighs 72 N on the surface of the earth. What is the gravitational force on it, at a height equal to half the radius of the earth?

(a) 48 N (b) 32 N
(c) 30 N (d) 24 N *(NEET 2020)*

27. A body weighs 200 N on the surface of the earth. How much will it weigh half way down to the centre of the earth ?

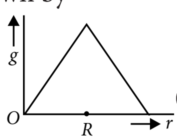
(a) 100 N (b) 150 N
(c) 200 N (d) 250 N *(NEET 2019)*

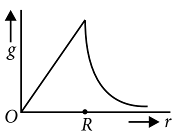
28. The acceleration due to gravity at a height 1 km above the earth is the same as at a depth d below the surface of earth. Then

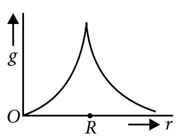
(a) $d = 1$ km (b) $d = \frac{3}{2}$ km

(c) $d = 2$ km (d) $d = \frac{1}{2}$ km *(NEET 2017)*

29. Starting from the centre of the earth having radius R, the variation of g (acceleration due to gravity) is shown by

(a)


(b)


(c)


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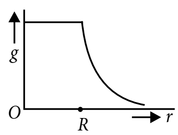
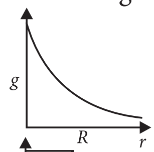


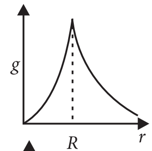
(NEET-II 2016)

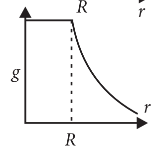
30. The height at which the weight of a body becomes $\left(\frac{1}{16}\right)$th, its weight on the surface of earth (radius R), is

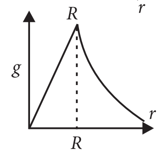
(a) $5R$ (b) $15R$ (c) $3R$ (d) $4R$ *(2012)*

31. The dependence of acceleration due to gravity g on the distance r from the centre of the earth, assumed to be a sphere of radius R and of uniform density is as shown in figures.

(1)


(2)


(3)


(4)


The correct figure is

(a) (4) (b) (1) (c) (2) (d) (3)
(Mains 2010)

8.7 Gravitational Potential Energy

32. The work done to raise a mass m from the surface of the earth to a height h, which is equal to the radius of the earth, is

(a) $\frac{3}{2}mgR$ (b) mgR

(c) $2mgR$ (d) $\frac{1}{2}mgR$ *(NEET 2019)*

33. At what height from the surface of earth the gravitation potential and the value of g are -5.4×10^7 J kg^{-1} and 6.0 m s^{-2} respectively? Take the radius of earth as 6400 km.

(a) 1400 km (b) 2000 km
(c) 2600 km (d) 1600 km *(NEET-I 2016)*

34. Infinite number of bodies, each of mass 2 kg are situated on x-axis at distances 1 m, 2 m, 4 m, 8 m, ..., respectively, from the origin. The resulting gravitational potential due to this system at the origin will be

(a) $-\frac{4}{3}G$ (b) $-4G$ (c) $-G$ (d) $-\frac{8}{3}G$
(NEET 2013)

35. A body of mass 'm' is taken from the earth's surface to the height equal to twice the radius (R) of the earth. The change in potential energy of body will be

(a) $3mgR$ (b) $\frac{1}{3}mgR$

(c) $mg\,2R$ (d) $\frac{2}{3}mgR$ *(NEET 2013)*

36. A particle of mass M is situated at the centre of a spherical shell of same mass and radius a. The

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magnitude of the gravitational potential at a point situated at $a/2$ distance from the centre, will be

(a) $\frac{GM}{a}$ (b) $\frac{2GM}{a}$ (c) $\frac{3GM}{a}$ (d) $\frac{4GM}{a}$ *(Mains 2011, 2010)*

37. A body of mass m is placed on earth's surface which is taken from earth surface to a height of $h = 3R$, then change in gravitational potential energy is

(a) $\frac{mgR}{4}$ (b) $\frac{2}{3}mgR$
(c) $\frac{3}{4}mgR$ (d) $\frac{mgR}{2}$ *(2003)*

8.8 Escape Speed

38. The ratio of escape velocity at earth (v_e) to the escape velocity at a planet (v_p) whose radius and mean density are twice as that of earth is

(a) 1 : 4 (b) $1:\sqrt{2}$ (c) 1 : 2 (d) $1:2\sqrt{2}$ *(NEET-I 2016)*

39. A black hole is an object whose gravitational field is so strong that even light cannot escape from it. To what approximate radius would earth (mass $= 5.98 \times 10^{24}$ kg) have to be compressed to be a black hole?

(a) 10^{-9} m (b) 10^{-6} m
(c) 10^{-2} m (d) 100 m *(2014)*

40. The radius of a planet is twice the radius of earth. Both have almost equal average mass-densities. V_P and V_E are escape velocities of the planet and the earth, respectively, then

(a) $V_P = 1.5\ V_E$ (b) $V_P = 2\ V_E$
(c) $V_E = 3\ V_P$ (d) $V_E = 1.5\ V_P$ *(Karnataka NEET 2013)*

41. A particle of mass 'm' is kept at rest at a height '$3R$' from the surface of earth, where 'R' is radius of earth and 'M' is mass of earth. The minimum speed with which it should be projected, so that it does not return back, is (g is acceleration due to gravity on the surface of earth)

(a) $\left(\frac{GM}{2R}\right)^{1/2}$ (b) $\left(\frac{gR}{4}\right)^{1/2}$
(c) $\left(\frac{2g}{R}\right)^{1/2}$ (d) $\left(\frac{GM}{R}\right)^{1/2}$

(Karnataka NEET 2013)

42. A particle of mass m is thrown upwards from the surface of the earth, with a velocity u. The mass and the radius of the earth are, respectively, M and R. G is gravitational constant and g is acceleration due to gravity on the surface of the earth. The minimum value of u so that the particle does not return back to earth, is

(a) $\sqrt{\frac{2GM}{R^2}}$ (b) $\sqrt{\frac{2GM}{R}}$
(c) $\sqrt{\frac{2gM}{R^2}}$ (d) $\sqrt{2gR^2}$ *(Mains 2011)*

43. The earth is assumed to be a sphere of radius R. A platform is arranged at a height R from the surface of the earth. The escape velocity of a body from this platform is fv, where v is its escape velocity from the surface of the Earth. The value of f is

(a) 1/2 (b) $\sqrt{2}$
(c) $1/\sqrt{2}$ (d) 1/3 *(2006)*

44. With what velocity should a particle be projected so that its height becomes equal to radius of earth?

(a) $\left(\frac{GM}{R}\right)^{1/2}$ (b) $\left(\frac{8GM}{R}\right)^{1/2}$
(c) $\left(\frac{2GM}{R}\right)^{1/2}$ (d) $\left(\frac{4GM}{R}\right)^{1/2}$ *(2001)*

45. For a planet having mass equal to mass of the earth but radius is one fourth of radius of the earth. The escape velocity for this planet will be

(a) 11.2 km/s (b) 22.4 km/s
(c) 5.6 km/s (d) 44.8 km/s *(2000)*

46. The escape velocity of a sphere of mass m is given by (G = Universal gravitational constant; M_e = Mass of the earth and R_e = Radius of the earth)

(a) $\sqrt{\frac{2GM_e m}{R_e}}$ (b) $\sqrt{\frac{2GM_e}{R_e}}$
(c) $\sqrt{\frac{GM_e}{R_e}}$ (d) $\sqrt{\frac{2GM_e + R_e}{R_e}}$ *(1999)*

47. The escape velocity of a body on the surface of the earth is 11.2 km/s. If the earth's mass increases to twice its present value and radius of the earth becomes half, the escape velocity becomes

(a) 22.4 km/s (b) 44.8 km/s
(c) 5.6 km/s (d) 11.2 km/s *(1997)*

48. The escape velocity from earth is 11.2 km/s. If a body is to be projected in a direction making an angle 45° to the vertical, then the escape velocity is

(a) 11.2×2 km/s (b) 11.2 km/s
(c) $11.2/\sqrt{2}$ km/s (d) $11.2\sqrt{2}$ km/s *(1993)*

49. For a satellite escape velocity is 11 km/s. If the satellite is launched at an angle of 60° with the vertical, then escape velocity will be

(a) 11 km/s (b) $11\sqrt{3}$ km/s
(c) $\frac{11}{\sqrt{3}}$ km/s (d) 33 km/s *(1989)*

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8.9 Earth Satellite

50. The time period of a geostationary satellite is 24 h, at a height $6R_E$ (R_E is radius of earth) from surface of earth. The time period of another satellite whose height is 2.5 R_E from surface will be,

(a) $6\sqrt{2}$ h (b) $12\sqrt{2}$ h

(c) $\frac{24}{2.5}$ h (d) $\frac{12}{2.5}$ h

(Odisha NEET 2019)

51. A remote-sensing satellite of earth revolves in a circular orbit at a height of 0.25×10^6 m above the surface of earth. If earth's radius is 6.38×10^6 m and $g = 9.8$ m s^{-2}, then the orbital speed of the satellite is

(a) 9.13 km s^{-1} (b) 6.67 km s^{-1}

(c) 7.76 km s^{-1} (d) 8.56 km s^{-1} *(2015)*

52. A satellite S is moving in an elliptical orbit around the earth. The mass of the satellite is very small compared to the mass of the earth. Then,

(a) the linear momentum of S remains constant in magnitude

(b) the acceleration of S is always directed towards the centre of the earth

(c) the angular momentum of S about the centre of the earth changes in direction, but its magnitude remains constant

(d) the total mechanical energy of S varies periodically with time *(2015)*

53. A geostationary satellite is orbiting the earth at a height of $5R$ above the surface of the earth, R being the radius of the earth. The time period of another satellite in hours at a height of $2R$ from the surface of the earth is

(a) 5 (b) 10 (c) $6\sqrt{2}$ (d) $\frac{6}{\sqrt{2}}$ *(2012)*

54. If v_e is escape velocity and v_o is orbital velocity of a satellite for orbit close to the earth's suface, then these are related by

(a) $v_o = \sqrt{2}v_e$ (b) $v_o = v_e$

(c) $v_e = \sqrt{2v_o}$ (d) $v_e = \sqrt{2}v_o$ *(Mains 2012)*

55. The radii of circular orbits of two satellites A and B of the earth, are $4R$ and R, respectively. If the speed of satellite A is $3V$, then the speed of satellite B will be

(a) $\frac{3V}{4}$ (b) $6V$

(c) $12V$ (d) $\frac{3V}{2}$ *(2010)*

56. A ball is dropped from a spacecraft revolving around the earth at a height of 120 km. What will happen to the ball?

(a) it will fall down to the earth gradually

(b) it will go very far in the space

(c) it will continue to move with the same speed along the original orbit of spacecraft

(d) it will move with the same speed, tangentially to the spacecraft. *(1996)*

57. A satellite A of mass m is at a distance of r from the centre of the earth. Another satellite B of mass $2m$ is at a distance of $2r$ from the earth's centre. Their time periods are in the ratio of

(a) 1 : 2 (b) 1 : 16

(c) 1 : 32 (d) $1:2\sqrt{2}$ *(1993)*

8.10 Energy of an Orbiting Satellite

58. A satellite of mass m is orbiting the earth (of radius R) at a height h from its surface. The total energy of the satellite in terms of g_0, the value of acceleration due to gravity at the earth's surface, is

(a) $\frac{mg_0R^2}{2(R+h)}$ (b) $-\frac{mg_0R^2}{2(R+h)}$

(c) $\frac{2mg_0R^2}{R+h}$ (d) $-\frac{2mg_0R^2}{R+h}$

(NEET-II 2016)

59. The additional kinetic energy to be provided to a satellite of mass m revolving around a planet of mass M, to transfer it from a circular orbit of radius R_1 to another of radius R_2 ($R_2 > R_1$) is

(a) $GmM\left(\frac{1}{R_1^2}-\frac{1}{R_2^2}\right)$

(b) $GmM\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

(c) $2GmM\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

(d) $\frac{1}{2}GmM\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ *(Mains 2010)*

60. Two satellites of earth, S_1 and S_2 are moving in the same orbit. The mass of S_1 is four times the mass of S_2. Which one of the following statements is true?

(a) The potential energies of earth and satellite in the two cases are equal.

(b) S_1 and S_2 are moving with the same speed.

(c) The kinetic energies of the two satellites are equal.

(d) The time period of S_1 is four times that of S_2. *(2007)*

61. For a satellite moving in an orbit around the earth, the ratio of kinetic energy to potential energy is

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(a) 1/2 (b) $1/\sqrt{2}$
(c) 2 (d) $\sqrt{2}$ *(2005)*

62. The satellite of mass m is orbiting around the earth in a circular orbit with a velocity v. What will be its total energy ?
(a) $(3/4)mv^2$ (b) $(1/2)mv^2$
(c) mv^2 (d) $-(1/2)mv^2$ *(1991)*

8.11 Geostationary and Polar Satellites

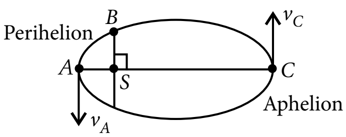
63. The mean radius of earth is R, its angular speed on its own axis is ω and the acceleration due to gravity at earth's surface is g. What will be the radius of the orbit of a geostationary satellite ?
(a) $(R^2g/\omega^2)^{1/3}$ (b) $(Rg/\omega^2)^{1/3}$
(c) $(R^2\omega^2/g)^{1/3}$ (d) $(R^2g/\omega)^{1/3}$ *(1992)*

ANSWER KEY

1.	(b)	**2.**	(b)	**3.**	(b)	**4.**	(a)	**5.**	(b)	**6.**	(c)	**7.**	(c)	**8.**	(a)	**9.**	(d)	**10.**	(a)
11.	(b)	**12.**	(b)	**13.**	(b)	**14.**	(b)	**15.**	(a)	**16.**	(a)	**17.**	(b)	**18.**	(d)	**19.**	(a)	**20.**	(d)
21.	(d)	**22.**	(b)	**23.**	(b)	**24.**	(a)	**25.**	(c)	**26.**	(b)	**27.**	(a)	**28.**	(c)	**29.**	(b)	**30.**	(c)
31.	(a)	**32.**	(d)	**33.**	(c)	**34.**	(b)	**35.**	(d)	**36.**	(c)	**37.**	(c)	**38.**	(d)	**39.**	(c)	**40.**	(b)
41.	(a)	**42.**	(b)	**43.**	(c)	**44.**	(a)	**45.**	(b)	**46.**	(b)	**47.**	(a)	**48.**	(b)	**49.**	(a)	**50.**	(a)
51.	(c)	**52.**	(b)	**53.**	(c)	**54.**	(d)	**55.**	(b)	**56.**	(c)	**57.**	(d)	**58.**	(b)	**59.**	(d)	**60.**	(b)
61.	(a)	**62.**	(d)	**63.**	(a)														

Hints & Explanations

1. (b) :



Point A is perihelion and C is aphelion.
So, $v_A > v_B > v_C$
As kinetic energy $K = (1/2)\,mv^2$ or $K \propto v^2$
So, $K_A > K_B > K_C$.

2. (b) : According to the law of conservation of angular momentum $L_1 = L_2$

$$mv_1r_1 = mv_2r_2 \Rightarrow v_1r_1 = v_2r_2 \text{ or } \frac{v_1}{v_2} = \frac{r_2}{r_1}$$

3. (b) : Equal areas are swept in equal time.
As it is given that area $SCD = 2 \times$ area of SAB
The time taken to go from C to D, $t_1 = 2t_2$
where t_2 is the time taken to go from A to B.

4. (a) : Period of revolution of planet A, $(T_A) = 8T_B$. According to Kepler's III law of planetary motion $T^2 \propto R^3$.

$$\text{Therefore} = \left(\frac{r_A}{r_B}\right)^3 = \left(\frac{T_A}{T_B}\right)^2 = \left(\frac{8T_B}{T_B}\right)^2 = 64$$

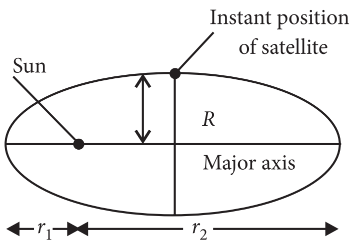
or $\frac{r_A}{r_B} = 4$ or $r_A = 4r_B$

5. (b) : Distance of two planets from sun, $r_1 = 10^{13}$ m and $r_2 = 10^{12}$ m
Relation between time period (T) and distance of the planet from the sun is $T^2 \propto r^3$ or $T \propto r^{3/2}$.

$$\text{Therefore, } \frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2} = \left(\frac{10^{13}}{10^{12}}\right)^{3/2} = 10^{3/2} = 10\sqrt{10}$$

6. (c) : In a circular or elliptical orbital motion a planet, angular momentum is conserved. In attractive field, potential energy and the total energy is negative. Kinetic energy increases with increase is velocity. If the motion is in a plane, the direction of L does not change.

7. (c) :



Applying the properties of ellipse, we have

$$\frac{2}{R} = \frac{1}{r_1} + \frac{1}{r_2} = \frac{r_1 + r_2}{r_1 r_2}\ ;\ R = \frac{2r_1r_2}{r_1 + r_2}$$

8. (a) : Since two astronauts are floating in gravitational free space. The only force acting on the two astronauts is the gravitational pull of their masses, $F = \frac{Gm_1m_2}{r^2}$, which is attractive in nature.
Hence they move towards each other.

9. (d) : Gravitational force of attraction between sun and planet provides centripetal force for the orbit of planet.

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$\therefore \quad \frac{GMm}{r^2} = \frac{mv^2}{r}; \ v^2 = \frac{GM}{r}$ ...(i)

Time period of the planet is given by

$$T = \frac{2\pi r}{v}, \ T^2 = \frac{4\pi^2 r^2}{v^2} = \frac{4\pi^2 r^2}{\left(\frac{GM}{r}\right)} \quad \text{(Using (i))}$$

$$T^2 = \frac{4\pi^2 r^3}{GM} \quad \text{...(ii)}$$

According to question, $T^2 = Kr^3$ ...(iii)

Comparing equations (ii) and (iii), we get

$$K = \frac{4\pi^2}{GM}, \ \therefore \ GMK = 4\pi^2$$

10. (a) : For a point inside the earth *i.e.* $r < R$

$$E = -\frac{GM}{R^3} r$$

where M and R be mass and radius of the earth respectively.

At the centre, $r = 0$

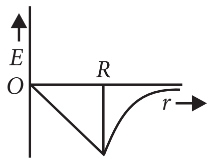
$\therefore \quad E = 0$

For a point outside the earth *i.e.* $r > R$,

$$E = -\frac{GM}{r^2}$$

On the surface of the earth *i.e.* $r = R$,

$$E = -\frac{GM}{R^2}$$



The variation of E with distance r from the centre is as shown in the figure.

11. (b) : Gravitational field due to the thin spherical shell

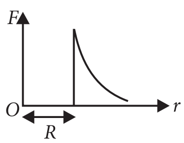
Inside the shell, (for $r < R$) $F = 0$

On the surface of the shell, (for $r = R$)

$$F = \frac{GM}{R^2}$$

Outside the shell, (for $r > R$)

$$F = \frac{GM}{r^2}$$



The variation of F with distance r from the centre is as shown in the figure.

12. (b) : The gravitational force does not depend upon the medium in which objects are placed.

13. (b)

14. (b) : $F_{surface} = G\frac{Mm}{R_e^2}$

$$F_{R_e/2} = G\frac{Mm}{(R_e + R_e/2)^2} = \frac{4}{9} \times F_{surface} = \frac{4}{9} \times 72 = 32 \text{ N}$$

15. (a) : Force between the two masses, $F = -G\frac{mm}{4R^2}$

This force will provide the necessary centripetal force for the masses to go around a circle, then

$$\frac{Gmm}{4R^2} = \frac{mv^2}{R} \ \Rightarrow v^2 = \frac{Gm}{4R} \ \Rightarrow v = \frac{1}{2}\sqrt{\frac{Gm}{R}}$$

16. (a) : Given : mass $(m) = 6 \times 10^{24}$ kg; angular velocity $(\omega) = 2 \times 10^{-7}$ rad/s and radius $(r) = 1.5 \times 10^8$ km $= 1.5 \times 10^{11}$ m

Force exerted on the earth $= mR\omega^2$

$= (6 \times 10^{24}) \times (1.5 \times 10^{11}) \times (2 \times 10^{-7})^2$

$= 36 \times 10^{21}$ N

17. (b) : Centripetal force $(F) = \frac{mv^2}{R}$ and the gravitational force $(F) = \frac{GMm}{R^2} = \frac{GMm}{R}$ (where $R^2 \to R$). Since $\frac{mv^2}{R} = \frac{GMm}{R}$ therefore $v = \sqrt{GM}$. Thus velocity v is independent of R.

18. (d) : If universal gravitational constant becomes ten times, then $G' = 10\,G$.

So, acceleration due to gravity increases. *i.e*, (d) is the wrong option.

19. (a) : Gravitational force acting on particle of mass m is

$$F = \frac{GM_p m}{(D_p/2)^2}$$

Acceleration due to gravity experienced by the particle is

$$g = \frac{F}{m} = \frac{GM_p}{(D_p/2)^2} = \frac{4GM_p}{D_p^2}$$

20. (d) : $g = \frac{GM}{r^2} = \frac{G}{r^2}\left(\frac{4}{3} \times \pi r^3 \rho\right) = \frac{4}{3} \times \pi\rho G r$

$$\frac{g'}{g} = \frac{3R}{R} \ \Rightarrow \ g' = 3g$$

21. (d) : From equation of acceleration due to gravity.

$$g_e = \frac{GM_e}{R_e^2} = \frac{G(4/3)\pi R_e^3}{R_e^2}\rho_e$$

$$g_e \propto R_e \rho_e$$

Acceleration due to gravity of planet $g_p \propto R_p \rho_p$

$$R_e \rho_e = R_p \rho_p \ \Rightarrow \ R_e \rho_e = R_p 2\rho_e$$

$$\Rightarrow \ R_p = \frac{1}{2} R \qquad (\because R_e = R)$$

22. (b) : Initial velocity of the mass on both the planets is same.

i.e., $\sqrt{2g'h'} = \sqrt{2gh}$

$\sqrt{2 \times g' \times h'} = \sqrt{2 \times 9g' \times 2} \ \Rightarrow \ 2h' = 36$

$\Rightarrow \ h' = 18$ m

23. (b)

24. (a) : Acceleration due to gravity

$$g = G \times \frac{M}{R^2} = G\frac{(4/3)\pi R^3 \times \rho}{R^2} = G \times \frac{4}{3}\pi R \times \rho$$

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or $\rho = \dfrac{3g}{4\pi GR}$

25. (c) : Given : radius of earth (R_e) = 6400 km; radius of mars (R_m) = 3200 km; mass of earth (M_e) = 10 M_m and weight of the object on earth (W_e) = 200 N.

$$\frac{W_m}{W_e} = \frac{mg_m}{mg_e} = \frac{M_m}{M_e} \times \left(\frac{R_e}{R_m}\right)^2 = \frac{1}{10} \times (2)^2 = \frac{2}{5}$$

or $W_m = W_e \times \dfrac{2}{5} = 200 \times 0.4 = 80$ N

26. (b) : Gravitational force at a height h,

$$mg_h = \frac{mg_0}{\left(1+\frac{h}{r}\right)^2} = \frac{72}{\left(1+\frac{R/2}{R}\right)^2} \text{ or } mg_h = 32 \text{ N}$$

or $F_g = 32$ N

27. (a) : Acceleration due to gravity at a depth d,

$g_d = g\left(1-\dfrac{d}{R}\right)$

For $d = R/2 \Rightarrow g_d = g\left(1-\dfrac{R/2}{R}\right) = \dfrac{g}{2}$

Required weight $W' = mg_d = \dfrac{mg}{2} = \dfrac{W}{2} = \dfrac{200}{2} = 100$ N

28. (c) : The acceleration due to gravity at a height h is given as

$$g_h = g\left(1-\frac{2h}{R_e}\right)$$

where R_e is radius of earth.

The acceleration due to gravity at a depth d is given as

$$g_d = g\left(1-\frac{d}{R_e}\right)$$

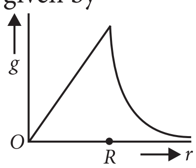
Given, $g_h = g_d$

$\therefore\ g\left(1-\dfrac{2h}{R_e}\right) = g\left(1-\dfrac{d}{R_e}\right)$ or, $d = 2h = 2 \times 1 = 2$ km

($\because h = 1$ km)

29. (b) : Acceleration due to gravity is given by

$$g = \begin{cases} \dfrac{4}{3}\pi\rho G r & ;\ r \le R \\ \dfrac{4}{3}\dfrac{\pi\rho R^3 G}{r^2} & ;\ r > R \end{cases}$$



30. (c) : Acceleration due to gravity at a height h from the surface of earth is

$$g' = \frac{g}{\left(1+\frac{h}{R}\right)^2} \quad \text{... (i)}$$

where g is the acceleration due to gravity at the surface of earth and R is the radius of earth.

Multiplying m (mass of the body) on both sides in (i), we get

$$mg' = \frac{mg}{\left(1+\frac{h}{R}\right)^2}$$

$\therefore$ Weight of body at height h, $W' = mg'$

Weight of body at surface of earth, $W = mg$

$\because\ W' = \dfrac{1}{16}W \quad \therefore\ \dfrac{1}{16} = \dfrac{1}{\left(1+\frac{h}{R}\right)^2}$

$\left(1+\dfrac{h}{R}\right)^2 = 16$ or $1+\dfrac{h}{R} = 4$

or $\dfrac{h}{R} = 3$ or $h = 3R$

31. (a)

32. (d) : Work done = Change in potential energy

$$= u_f - u_i = \frac{-GMm}{(R+h)} - \left(\frac{-GMm}{R}\right)$$

where M is the mass of earth and R is the radius of earth.

$\therefore\ W = GMm\left[\dfrac{1}{R} - \dfrac{1}{(R+h)}\right]$

Now, $h = R$

$\therefore\ W = GMm\left[\dfrac{1}{R} - \dfrac{1}{2R}\right] = \dfrac{GMm}{2R}$

$\Rightarrow\ W = \dfrac{mgR}{2} \qquad \left[\because\ g = \dfrac{GM}{R^2}\right]$

33. (c) : Gravitation potential at a height h from the surface of earth, $V_h = -5.4 \times 10^7$ J kg^{-1}

At the same point acceleration due to gravity, $g_h = 6$ m s^{-2}

$R = 6400$ km $= 6.4 \times 10^6$ m

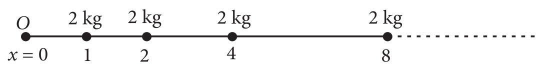
We know, $V_h = -\dfrac{GM}{(R+h)}$,

$$g_h = \frac{GM}{(R+h)^2} = -\frac{V_h}{R+h} \Rightarrow R+h = -\frac{V_h}{g_h}$$

$$\therefore\ h = -\frac{V_h}{g_h} - R = -\frac{(-5.4\times10^7)}{6} - 6.4\times10^6$$

$= 9 \times 10^6 - 6.4 \times 10^6 = 2600$ km

34. (b) : The resulting gravitational potential at the origin O due to each of mass 2 kg located at positions as shown in figure is



$$V = -\frac{G\times2}{1} - \frac{G\times2}{2} - \frac{G\times2}{4} - \frac{G\times2}{8} - ...$$

$$= -2G\left[1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+......\right] = -2G\left[\frac{1}{1-\frac{1}{2}}\right]$$

$$= -2G\left[\frac{2}{1}\right] = -4G$$

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35. (d) : Gravitational potential energy at any point at a distance r from the centre of the earth is

$$U = -\frac{GMm}{r}$$

where M and m be masses of the earth and the body respectively.

At the surface of the earth, $r = R;\ U_i = -\frac{GMm}{R}$

At a height h from the surface,

$r = R + h = R + 2R = 3R$ ($h = 2R$ (Given))

$$\therefore\ U_f = -\frac{GMm}{3R}$$

Change in potential energy, $\Delta U = U_f - U_i$

$$= -\frac{GMm}{3R} - \left(-\frac{GMm}{R}\right) = \frac{GMm}{R}\left(1 - \frac{1}{3}\right)$$

$$= \frac{2}{3}\frac{GMm}{R} = \frac{2}{3}mgR \qquad \left(\because g = \frac{GM}{R^2}\right)$$

36. (c) : Here, Mass of a particle = M

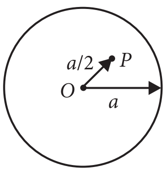
Mass of a spherical shell = M

Radius of a spherical shell = a

Let O be centre of a spherical shell.

Gravitational potential at point P due to particle at O is

$$V_1 = -\frac{GM}{a/2}$$



Gravitational potential at point P due to spherical shell is

$$V_2 = -\frac{GM}{a}$$

Hence, total gravitational potential at point P is

$$V = V_1 + V_2$$

$$= \frac{-GM}{a/2} + \left(\frac{-GM}{a}\right) = \frac{-2GM}{a} - \frac{GM}{a} = \frac{-3GM}{a}$$

$$|V| = \frac{3GM}{a}$$

37. (c) : Gravitational potential energy on earth's surface $= -\frac{GMm}{R}$, where M and R are the mass and radius of the earth respectively, m is the mass of the body and G is the universal gravitational constant.

Gravitational potential energy at a height $h = 3R$

$$= -\frac{GMm}{R+h} = -\frac{GMm}{R+3R} = -\frac{GMm}{4R}$$

$\therefore$ Change in potential energy

$$= -\frac{GMm}{4R} - \left(-\frac{GMm}{R}\right)$$

$$= -\frac{GMm}{4R} + \frac{GMm}{R} = \frac{3}{4}\frac{GMm}{R} = \frac{3}{4}mgR$$

38. (d) : As escape velocity, $v = \sqrt{\frac{2GM}{R}}$

$$= \sqrt{\frac{2G}{R}\cdot\frac{4\pi R^3}{3}\rho} = R\sqrt{\frac{8\pi G}{3}\rho}$$

$$\therefore\ \frac{v_e}{v_p} = \frac{R_e}{R_p} \times \sqrt{\frac{\rho_e}{\rho_p}} = \frac{1}{2} \times \sqrt{\frac{1}{2}} = \frac{1}{2\sqrt{2}}$$

($\because\ R_p = 2R_e$ and $\rho_p = 2\rho_e$)

39. (c) : Light cannot escape from a black hole,

$$v_e = c \Rightarrow \sqrt{\frac{2GM}{R}} = c \quad \text{or} \quad R = \frac{2GM}{c^2}$$

$$R = \frac{2 \times 6.67 \times 10^{-11}\ \text{N m}^2\text{kg}^{-2} \times 5.98 \times 10^{24}\ \text{kg}}{(3 \times 10^8\ \text{m s}^{-1})^2}$$

$= 8.86 \times 10^{-3}$ m $\propto 10^{-2}$ m

40. (b) : Here, $R_P = 2R_E$, $\rho_E = \rho_P$

Escape velocity of the earth,

$$V_E = \sqrt{\frac{2GM_E}{R_E}} = \sqrt{\frac{2G}{R_E}\left(\frac{4}{3}\pi R_E^3 \rho_E\right)} = R_E\sqrt{\frac{8}{3}\pi G\,\rho_E} \qquad \ldots\text{(i)}$$

Escape velocity of the planet,

$$V_P = \sqrt{\frac{2GM_P}{R_P}} = \sqrt{\frac{2G}{R_P}\left(\frac{4}{3}\pi R_P^3 \rho_P\right)} = R_P\sqrt{\frac{8}{3}\pi G\,\rho_P} \qquad \ldots\text{(ii)}$$

Divide (i) by (ii), we get

$$\frac{V_E}{V_P} = \frac{R_E}{R_P}\sqrt{\frac{\rho_E}{\rho_P}} = \frac{R_E}{2R_E}\sqrt{\frac{\rho_E}{\rho_E}} = \frac{1}{2} \quad \text{or} \quad V_P = 2V_E$$

41. (a) : The minimum speed with which the particle should be projected from the surface of the earth so that it does not return back is known as escape speed and it is given by

$$v_e = \sqrt{\frac{2GM}{(R+h)}}$$

Here, $h = 3R$

$$\therefore\ v_e = \sqrt{\frac{2GM}{(R+3R)}} = \sqrt{\frac{2GM}{4R}} = \sqrt{\frac{GM}{2R}}$$

42. (b) : According to law of conservation of mechanical energy,

$$\frac{1}{2}mu^2 - \frac{GMm}{R} = 0 \quad \text{or} \quad u^2 = \frac{2GM}{R}$$

$$u = \sqrt{\frac{2GM}{R}} = \sqrt{2gR} \qquad \left(\because g = \frac{GM}{R^2}\right)$$

43. (c) : Escape velocity of the body from the surface of earth is $v = \sqrt{2gR}$ ...(i)

For escape velocity of the body from the platform, potential energy + kinetic energy = 0

$$-\frac{GMm}{2R} + \frac{1}{2}mv_e^2 = 0$$

$$\Rightarrow\ v_e = \sqrt{\frac{GM}{R^2}\cdot R} = \sqrt{gR} \Rightarrow fv = \sqrt{gR} \qquad \text{...(ii)}$$

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From equation (i) and (ii), we get $f = \frac{1}{\sqrt{2}}$

44. (a) : Use $v^2 = \frac{2gh}{1+\frac{h}{R}}$ given $h = R$.

$\therefore \ v = \sqrt{gR} = \sqrt{\frac{GM}{R}}$

45. (b) : $v_e = \sqrt{2gR_e} = \sqrt{\frac{2GM}{R_e}}$

$\because R_P = \frac{1}{4}R_e \ \ v_P = 2v_e = 2 \times 11.2 = 22.4$ km/s.

46. (b)

47. (a) : Escape velocity of a body $(v_e) = 11.2$ km/s; New mass of the earth $M'_e = 2M_e$ and new radius of the earth $R'_e = 0.5\ R_e$.

Escape velocity $(v_e) = \sqrt{\frac{2GM_e}{R_e}} \propto \sqrt{\frac{M_e}{R_e}}$

Therefore $\frac{v_e}{v'_e} = \sqrt{\frac{M_e}{R_e} \times \frac{0.5R_e}{2M_e}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

or, $v'_e = 2v_e = 22.4$ km/s

48. (b) : Escape velocity does not depend on the angle of projection.

49. (a)

50. (a) : Time period of Geostationary satellite is,

$$T = 2\pi\sqrt{\frac{a^3}{GM}} \quad \Rightarrow \quad T^2 \propto a^3$$

$$\therefore \ \frac{T_1^2}{T_2^2} = \frac{a_1^3}{a_2^3} \quad \Rightarrow \quad \frac{(24)^2}{T_2^2} = \frac{(7R_E)^3}{(3.5R_E)^3}$$

$$\Rightarrow \ T_2^2 = \frac{(24)^2 \times (3.5)^3}{(7)^3} \ \Rightarrow \ T_2 = \frac{\sqrt{(24)^2}}{\sqrt{8}} = 6\sqrt{2}\text{ h.}$$

51. (c) : The orbital speed of the satellite is,

$$v_o = R\sqrt{\frac{g}{(R+h)}}$$

where R is the earth's radius, g is the acceleration due to gravity on earth's surface and h is the height above the surface of earth.

Here, $R = 6.38 \times 10^6$ m, $g = 9.8$ m s^{-2}, $h = 0.25 \times 10^6$ m

$$\therefore \ v_o = (6.38\times10^6\text{ m})\sqrt{\frac{(9.8\text{ m s}^{-2})}{(6.38\times10^6\text{ m}+0.25\times10^6\text{ m})}}$$

$$= 7.76 \times 10^3 \text{ m s}^{-1} = 7.76 \text{ km s}^{-1}$$

52. (b) : The gravitational force on the satellite S acts towards the centre of the earth, so the acceleration of the satellite S is always directed towards the centre of the earth.

53. (c) : According to Kepler's third law $T \propto r^{3/2}$

$$\therefore \ \frac{T_2}{T_1} = \left(\frac{r_2}{r_1}\right)^{3/2} = \left(\frac{R+2R}{R+5R}\right)^{3/2} = \frac{1}{2^{3/2}}$$

Since $T_1 = 24$ hours so,

$$\frac{T_2}{24} = \frac{1}{2^{3/2}} \ \text{ or } \ T_2 = \frac{24}{2^{3/2}} = \frac{24}{2\sqrt{2}} = 6\sqrt{2}\text{ hours}$$

54. (d) : Escape velocity, $v_e = \sqrt{\frac{2GM}{R}}$ ...(i)

where M and R be the mass and radius of the earth respectively.

The orbital velocity of a satellite close to the earth's surface is

$$v_o = \sqrt{\frac{GM}{R}} \quad \text{...(ii)}$$

From (i) and (ii), we get $v_e = \sqrt{2}v_o$

55. (b) : Orbital speed of the satellite around the earth is

$$v = \sqrt{\frac{GM}{r}}$$

For satellite A, $r_A = 4R$, $v_A = 3V$

$$v_A = \sqrt{\frac{GM}{r_A}} \quad \text{...(i)}$$

For satellite B, $r_B = R$, $v_B = ?$

$$v_B = \sqrt{\frac{GM}{r_B}} \quad \text{...(ii)}$$

Dividing equation (ii) by equation (i), we get

$$\frac{v_B}{v_A} = \sqrt{\frac{r_A}{r_B}} \ \text{ or } \ v_B = v_A\sqrt{\frac{r_A}{r_B}}$$

Substituting the given values, we get

$$v_B = 3V\sqrt{\frac{4R}{R}} \ \text{ or } \ v_B = 6V$$

56. (c) : Since no external torque is applied therefore, according to law of conservation of angular momentum, the ball will continue to move with the same angular velocity along the original orbit of the spacecraft.

57. (d) : Time period of satellite does not depend on its mass.

As $T^2 \propto r^3$

$$\frac{T_A}{T_B} = \frac{r^{3/2}}{2^{3/2}r^{3/2}} = \frac{1}{2\sqrt{2}}$$

58. (b) : Total energy of satellite at height h from the earth's surface,

$$E = PE + KE = -\frac{GMm}{(R+h)} + \frac{1}{2}mv^2 \quad \text{...(i)}$$

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Also, $\frac{mv^2}{(R+h)}=\frac{GMm}{(R+h)^2}$ or, $v^2=\frac{GM}{R+h}$ ...(ii)

From eqns. (i) and (ii),

$$E=-\frac{GMm}{(R+h)}+\frac{1}{2}\frac{GMm}{(R+h)}=-\frac{1}{2}\frac{GMm}{(R+h)}$$

$$=-\frac{1}{2}\frac{GM}{R^2}\times\frac{mR^2}{(R+h)}=-\frac{mg_0R^2}{2(R+h)}$$

59. (d)

60. (b) : The satellite of mass m is moving in a circular orbit of radius r.

$\therefore$ Kinetic energy of the satellite, $K=\frac{GMm}{2r}$ ... (i)

Potential energy of the satellite, $U=\frac{-GMm}{r}$ ... (ii)

Orbital speed of satellite, $v=\sqrt{\frac{GM}{r}}$ ... (iii)

Time-period of satellite,

$$T=\left[\left(\frac{4\pi^2}{GM}\right)r^3\right]^{1/2} \quad \text{... (iv)}$$

Given $m_{S_1}=4m_{S_2}$

Since M, r is same for both the satellites S_1 and S_2.

$\therefore$ From equation (ii), we get $U\propto m$

$\therefore \quad \frac{U_{S_1}}{U_{S_2}}=\frac{m_{S_1}}{m_{S_2}}=4$ or, $U_{S_1}=4U_{S_2}$.

Option (a) is wrong.

From (iii), since v is independent of the mass of a satellite, the orbital speed is same for both satellites S_1 and S_2. Hence option (b) is correct.

From (i), we get $K\propto m$

$\therefore \quad \frac{K_{S_1}}{K_{S_2}}=\frac{m_{S_1}}{m_{S_2}}=4$ or, $K_{S_1}=4K_{S_2}$.

Hence option (c) is wrong.

From (iv), since T is independent of the mass of a satellite, time period is same for both the satellites S_1 and S_2. Hence option (d) is wrong.

61. (a) : $K.E.=\frac{GMm}{2R}$; $P.E.=-\frac{GMm}{R}$

$\therefore \quad K.E.=\frac{|P.E.|}{2}$ or, $\frac{K.E.}{|P.E.|}=\frac{1}{2}$

62. (d) : Total energy = –K.E. $=-\frac{1}{2}mv^2$

63. (a) : $\frac{GMm}{r^2}=m\omega^2r \Rightarrow r^3=\frac{GM}{\omega^2}=\frac{gR^2}{\omega^2}$

$\therefore \quad r=(gR^2/\omega^2)^{1/3}$.

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